Callable Bond

Callable Bond Introduction

- A callable bond is a bond in which the issuer has the right to call the bond at specified times (callable dates) from the investor for a specified price (call price).
- At each callable date prior to the bond maturity, the issuer may recall the bond from its investor by returning the investor's money.
- A callable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.
- Callable bonds protect issuers. Therefore, a callable bond normally pays the investor a higher coupon than a non-callable bond.
- The underlying bond can be a fixed rate bond or a floating rate bond.

Callable Bond Introduction (Cont.)

- Although a callable bond is a higher cost to the issuer and an uncertainty to the investor comparing to a regular bond, it is actually quite attractive to both issuers and investors.
- For issuers, callable bonds allow them to reduce interest costs at a future date should rate decrease.
- For investors, callable bonds allow them to earn a higher interest rate of return until the bonds are called off.
- If interest rates have declined since the issuer first issues the bond, the issuer is like to call its current bond and reissues it at a lower coupon.

Callable Bond

Callable Bond Payoffs

- At the bond maturity *T*, the payoff of a callable bond is given by

$$V_c(t) = \begin{cases} F + C & if\ not\ called \\ \min(P_c, F + C) & if\ called \end{cases}$$

where *F* – the principal or face value; *C* – the coupon; P_c – the call price; *min* (*x*, *y*) – the minimum of *x* and *y*

- The payoff of the callable bond at any call date T_i can be expressed as

$$V_c(T_i) = \begin{cases} \overline{V}_{T_i} & if\ not\ called \\ \min\left(P_c, \overline{V}_{T_i}\right) & if\ called \end{cases}$$

where $\overline{V}_{T_i}$ – continuation value at T_i

Model Selection

- Given the valuation complexity of callable bonds, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numerical solution to price them numerically.
- The selection of interest rate term structure models
  - Popular interest rate term structure models:

    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.

Model Selection (Cont.)

- The selection of numeric approaches
  - After selecting a term structure model, we need to choose a numerical approach to approximate the underlying stochastic process of the model.
  - Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - Therefore, we choose either PDE or lattice.
- Our decision is to use LGM plus lattice.

LGM Model

- The dynamics

$$dX(t) = \alpha(t)dW$$

where X is the single state variable and W is the Wiener process.

- The numeraire is given by

$$N(t,X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

- The zero coupon bond price is

$$B(t,X;T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

LGM Model (Cont.)

- The LGM model is mathematically equivalent to the Hull-White model but offers
  - Significant improvement of stability and accuracy for calibration.
  - Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.

LGM Model (Cont.)

- Match today's curve

  At time t=0, X(0)=0 and H(0)=0. Thus Z(0,0;T)=D(T). In other words, the LGM automatically fits today's discount curve.

- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

LGM Model (Cont.)

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the payoff of the callable bond at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the callable bond.

Callabe Bond

Example

Bond specification		Callable schedule	
Buy Sell	Buy	Call Price	Notification Date
Calendar	NYC	100	1/26/2015
Coupon Type	Fixed	100	7/25/2018
Currency	USD		
First Coupon Date	7/30/2013		
Interest Accrual Date	1/30/2013		
Issue Date	1/30/2013		
Last Coupon Date	1/30/2018		
Maturity Date	7/30/2018		
Settlement Lag	1		
Face Value	100		
Pay Receive	Receive		
Day Count	dc30360		
Payment Frequency	6		
Coupon	0.015		

Reference:
https://finpricing.com/lib/EqConvertible.html