Callable Bond
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Callable Bond Introduction

◆ A callable bond is a bond in which the issuer has the right to call the bond at specified times (callable dates) from the investor for a specified price (call price).

◆ At each callable date prior to the bond maturity, the issuer may recall the bond from its investor by returning the investor’s money.

◆ A callable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.

◆ Callable bonds protect issuers. Therefore, a callable bond normally pays the investor a higher coupon than a non-callable bond.

◆ The underlying bond can be a fixed rate bond or a floating rate bond.
Callable Bond Introduction (Cont.)

- Although a callable bond is a higher cost to the issuer and an uncertainty to the investor comparing to a regular bond, it is actually quite attractive to both issuers and investors.

- For issuers, callable bonds allow them to reduce interest costs at a future date should rates decrease.

- For investors, callable bonds allow them to earn a higher interest rate of return until the bonds are called off.

- If interest rates have declined since the issuer first issues the bond, the issuer is likely to call its current bond and reissues it at a lower coupon.
Callable Bond Payoffs

◆ At the bond maturity $T$, the payoff of a callable bond is given by

$$V_c(t) = \begin{cases} F + C & \text{if not called} \\ \min(P_c, F + C) & \text{if called} \end{cases}$$

where $F$ – the principal or face value; $C$ – the coupon; $P_c$ – the call price; $\min(x, y)$ – the minimum of $x$ and $y$

◆ The payoff of the callable bond at any call date $T_i$ can be expressed as

$$V_c(T_i) = \begin{cases} \bar{V}_{T_i} & \text{if not called} \\ \min(P_c, \bar{V}_{T_i}) & \text{if called} \end{cases}$$

where $\bar{V}_{T_i}$ – continuation value at $T_i$
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Model Selection

Given the valuation complexity of callable bonds, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numerical solution to price them numerically.

The selection of interest rate term structure models

- Popular interest rate term structure models:
  - Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.
The selection of numeric approaches

- After selecting a term structure model, we need to choose a numerical approach to approximate the underlying stochastic process of the model.
- Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
- Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
- Therefore, we choose either PDE or lattice.

Our decision is to use LGM plus lattice.
The dynamics

\[ dX(t) = \alpha(t)dW \]

where \( X \) is the single state variable and \( W \) is the Wiener process.

The numeraire is given by

\[ N(t,X) = \left( H(t)X + 0.5H^2(t)\zeta(t) \right)/D(t) \]

The zero coupon bond price is

\[ B(t,X;T) = D(T)\exp\left( -H(t)X - 0.5H^2(t)\zeta(t) \right) \]
LGM Model (Cont.)

- The LGM model is mathematically equivalent to the Hull-White model but offers
  - Significant improvement of stability and accuracy for calibration.
  - Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfectly correlated.
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LGM Model (Cont.)

- Match today’s curve
  At time $t=0$, $X(0)=0$ and $H(0)=0$. Thus $Z(0,0;T)=D(T)$. In other words, the LGM automatically fits today’s discount curve.
- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.
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LGM Model (Cont.)

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the payoff of the callable bond at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the callable bond.
## Example

### Bond specification

<table>
<thead>
<tr>
<th>Bond specification</th>
<th>Callable schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy Sell</td>
<td>Call Price</td>
</tr>
<tr>
<td>Calendar</td>
<td>Notification Date</td>
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<tr>
<td>Coupon Type</td>
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<tr>
<td>Currency</td>
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<tr>
<td>First Coupon Date</td>
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<tr>
<td>Interest Accrual Date</td>
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</tr>
<tr>
<td>Issue Date</td>
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</tr>
<tr>
<td>Last Coupon Date</td>
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<tr>
<td>Maturity Date</td>
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<tr>
<td>Settlement Lag</td>
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<tr>
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<td>Pay Receive</td>
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<tr>
<td>Coupon</td>
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</tbody>
</table>

- **Buy**: Buy
- **Sell**: Sell
- **Calendar**: NYC
- **Coupon Type**: Fixed
- **Currency**: USD
- **Interest Accrual Date**: 1/30/2013
- **Issue Date**: 1/30/2013
- **Last Coupon Date**: 1/30/2018
- **Maturity Date**: 7/30/2018
- **Settlement Lag**: 1
- **Face Value**: 100
- **Pay Receive**: Receive
- **Day Count**: dc30360
- **Payment Frequency**: 6
- **Coupon**: 0.015
Reference:
https://finpricing.com/lib/EqConvertible.html